

**2013 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS**

6. Consider the differential equation  $\frac{dy}{dx} = e^y(3x^2 - 6x)$ . Let  $y = f(x)$  be the particular solution to the differential equation that passes through  $(1, 0)$ .
- (a) Write an equation for the line tangent to the graph of  $f$  at the point  $(1, 0)$ . Use the tangent line to approximate  $f(1.2)$ .
- (b) Find  $y = f(x)$ , the particular solution to the differential equation that passes through  $(1, 0)$ .
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**STOP**

**END OF EXAM**

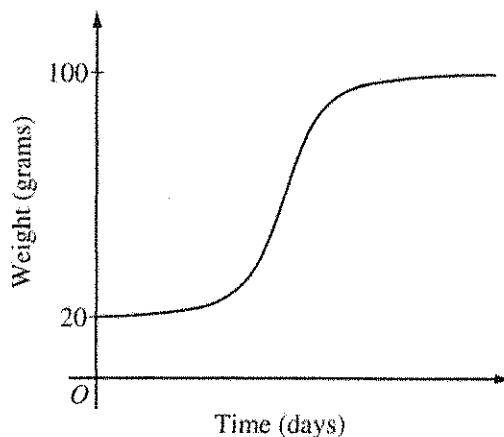
2012 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS

5. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time  $t = 0$ , when the bird is first weighed, its weight is 20 grams. If  $B(t)$  is the weight of the bird, in grams, at time  $t$  days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let  $y = B(t)$  be the solution to the differential equation above with initial condition  $B(0) = 20$ .

- (a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.
- (b) Find  $\frac{d^2B}{dt^2}$  in terms of  $B$ . Use  $\frac{d^2B}{dt^2}$  to explain why the graph of  $B$  cannot resemble the following graph.



- (c) Use separation of variables to find  $y = B(t)$ , the particular solution to the differential equation with initial condition  $B(0) = 20$ .

**2011 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS**

5. At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function  $W$  models the total amount of solid waste stored at the landfill. Planners estimate that  $W$  will satisfy the differential equation  $\frac{dW}{dt} = \frac{1}{25}(W - 300)$  for the next 20 years.  $W$  is measured in tons, and  $t$  is measured in years from the start of 2010.
- (a) Use the line tangent to the graph of  $W$  at  $t = 0$  to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time  $t = \frac{1}{4}$ ).
- (b) Find  $\frac{d^2W}{dt^2}$  in terms of  $W$ . Use  $\frac{d^2W}{dt^2}$  to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time  $t = \frac{1}{4}$ .
- (c) Find the particular solution  $W = W(t)$  to the differential equation  $\frac{dW}{dt} = \frac{1}{25}(W - 300)$  with initial condition  $W(0) = 1400$ .

**END OF EXAM**

2010 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS

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6. Solutions to the differential equation  $\frac{dy}{dx} = xy^3$  also satisfy  $\frac{d^2y}{dx^2} = y^3(1 + 3x^2y^2)$ . Let  $y = f(x)$  be a particular solution to the differential equation  $\frac{dy}{dx} = xy^3$  with  $f(1) = 2$ .
- (a) Write an equation for the line tangent to the graph of  $y = f(x)$  at  $x = 1$ .
  - (b) Use the tangent line equation from part (a) to approximate  $f(1.1)$ . Given that  $f(x) > 0$  for  $1 < x < 1.1$ , is the approximation for  $f(1.1)$  greater than or less than  $f(1.1)$ ? Explain your reasoning.
  - (c) Find the particular solution  $y = f(x)$  with initial condition  $f(1) = 2$ .
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WRITE ALL WORK IN THE PINK EXAM BOOKLET.

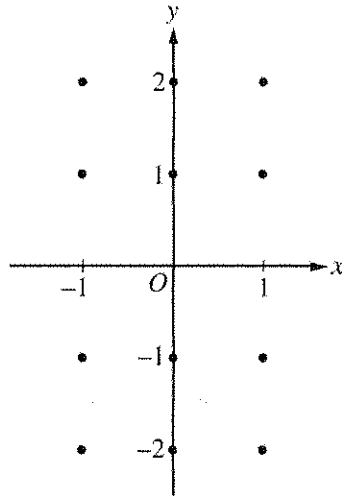
END OF EXAM

2010 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)

5. Consider the differential equation  $\frac{dy}{dx} = \frac{x+1}{y}$ .

- (a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and for  $-1 < x < 1$ , sketch the solution curve that passes through the point  $(0, -1)$ .

(Note: Use the axes provided in the exam booklet.)



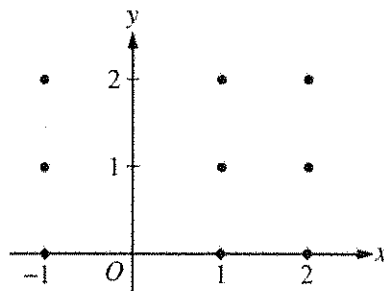
- (b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the  $xy$ -plane for which  $y \neq 0$ . Describe all points in the  $xy$ -plane,  $y \neq 0$ , for which  $\frac{dy}{dx} = -1$ .
- (c) Find the particular solution  $y = f(x)$  to the given differential equation with the initial condition  $f(0) = -2$ .

**2008 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS**

5. Consider the differential equation  $\frac{dy}{dx} = \frac{y-1}{x^2}$ , where  $x \neq 0$ .

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

(Note: Use the axes provided in the exam booklet.)



(b) Find the particular solution  $y = f(x)$  to the differential equation with the initial condition  $f(2) = 0$ .

(c) For the particular solution  $y = f(x)$  described in part (b), find  $\lim_{x \rightarrow \infty} f(x)$ .

**WRITE ALL WORK IN THE PINK EXAM BOOKLET.**

**END OF EXAM**

**2008 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)**

6. Consider the closed curve in the  $xy$ -plane given by

$$x^2 + 2x + y^4 + 4y = 5.$$

- (a) Show that  $\frac{dy}{dx} = \frac{-(x+1)}{2(y^3+1)}$ .
- (b) Write an equation for the line tangent to the curve at the point  $(-2, 1)$ .
- (c) Find the coordinates of the two points on the curve where the line tangent to the curve is vertical.
- (d) Is it possible for this curve to have a horizontal tangent at points where it intersects the  $x$ -axis?  
Explain your reasoning.
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**WRITE ALL WORK IN THE EXAM BOOKLET.**

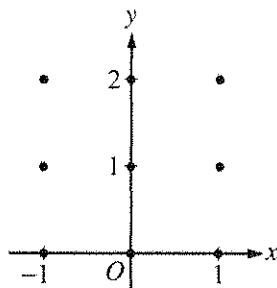
**END OF EXAM**

**2007 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)**

5. Consider the differential equation  $\frac{dy}{dx} = \frac{1}{2}x + y - 1$ .

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

(Note: Use the axes provided in the exam booklet.)



(b) Find  $\frac{d^2y}{dx^2}$  in terms of  $x$  and  $y$ . Describe the region in the  $xy$ -plane in which all solution curves to the differential equation are concave up.

(c) Let  $y = f(x)$  be a particular solution to the differential equation with the initial condition  $f(0) = 1$ . Does  $f$  have a relative minimum, a relative maximum, or neither at  $x = 0$ ? Justify your answer.

(d) Find the values of the constants  $m$  and  $b$ , for which  $y = mx + b$  is a solution to the differential equation.

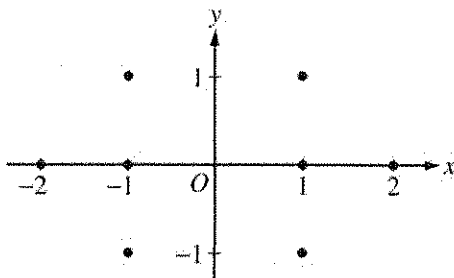
**WRITE ALL WORK IN THE EXAM BOOKLET.**

**END OF EXAM**



**2006 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS**

5. Consider the differential equation  $\frac{dy}{dx} = \frac{1+y}{x}$ , where  $x \neq 0$ .
- (a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.
- (Note: Use the axes provided in the pink exam booklet.)



- (b) Find the particular solution  $y = f(x)$  to the differential equation with the initial condition  $f(-1) = 1$  and state its domain.

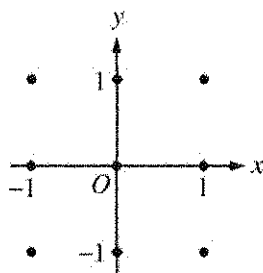
**END OF EXAM**

**2006 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)**

5. Consider the differential equation  $\frac{dy}{dx} = (y - 1)^2 \cos(\pi x)$ .

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

(Note: Use the axes provided in the exam booklet.)



(b) There is a horizontal line with equation  $y = c$  that satisfies this differential equation. Find the value of  $c$ .

(c) Find the particular solution  $y = f(x)$  to the differential equation with the initial condition  $f(1) = 0$ .

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